

# **The Eulerian and Lagrangian Predictability of Oceanic Flows**

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## **LONG-TERM GOALS**

We have two major goals. The first is to improve our capabilities and scientific knowledge concerning Lagrangian predictability in the ocean. Specific applications include search and rescue operations, the design of float and drifter experiments, and the understanding of stirring mechanisms. The second goal is to develop methods allowing the assessment of the underlying dynamics and predictability of complex physical systems in the ocean based solely on the analysis of time-series.

## **OBJECTIVES**

- (1) The immediate objective of work done over the past year has been to determine the relevance of chaotic advection in the ocean. Conventional turbulence and chaotic advection are the two major mechanisms that compete for control over horizontal fluid stirring. Knowledge of which mechanism dominates effects the way in which search and rescue is carried out.
- (2) With regard to the assessment of predictability using time-series analysis, our objective has been to develop methods of phase space reconstruction and surrogate time series analysis for use in connection with ocean data and ocean models.

## **APPROACH**

To achieve objective (1) we are using models capable of producing chaotic advection and/or turbulence. We are developing measures involving quantities such as kinetic energy spectra, passive tracer spectra, and relative dispersion in order to distinguish between the two regimes. We have also been seeking data and consulting with those observationalists who are in a position to perform the type of analysis that our studies suggest.

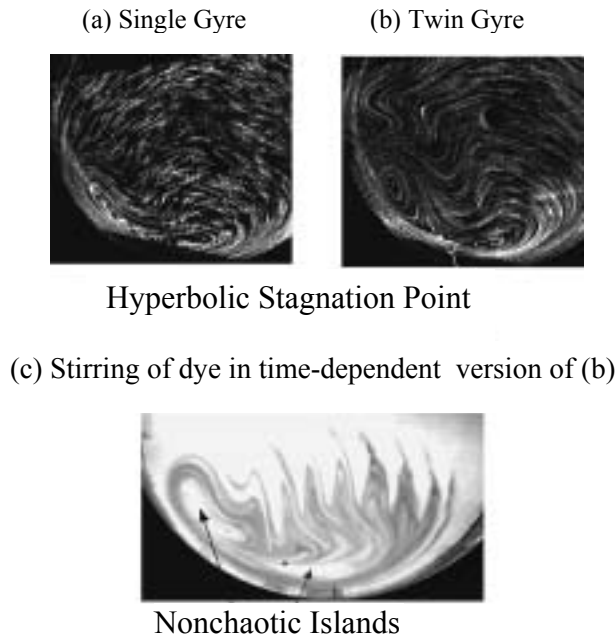
With regard to objective (2), we are testing methods of phase space reconstruction and surrogate time series analysis, both of which are relatively new to physical oceanography. The time series range in complexity from nearly periodic records obtained from models with known dynamics to strongly aperiodic records obtained from field programs. The tests often allow one to distinguish between linear and nonlinear systems, deterministic and stochastic systems, and chaotic vs. stochastic behavior. The measures also provide a measure of predictability.

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## WORK COMPLETED

With respect to the first objective mentioned above, we have identified four measures that help to distinguish between chaotic advection and turbulence. The measures include the rate of decay of wave number spectra of kinetic energy and passive tracer, the skewness of pdf's of float or drifter dispersion, and the ratio of Eulerian to Lagrangian time scales. These measures have emerged from past work by other authors and from analysis of our own numerical and laboratory models. We have identified a number of field programs and data sets that lend themselves to the type of analysis required. In some cases, the analysis has already been done by the investigators in question, though without knowledge of the implications for chaotic advection and predictability.

With respect to the time-series analysis, we have performed phase space reconstruction and surrogate time series analysis on a hierarchy of time-series. The simplest time series are generated by a numerical model of a time-dependent ocean overflow. Slightly more complicated is a noise-contaminated current meter record taken from the N. Carolina shelf area and dominated by tides. The most complicated time series come from moored current meters deployed near the shelf break front in the Middle Atlantic Bight.



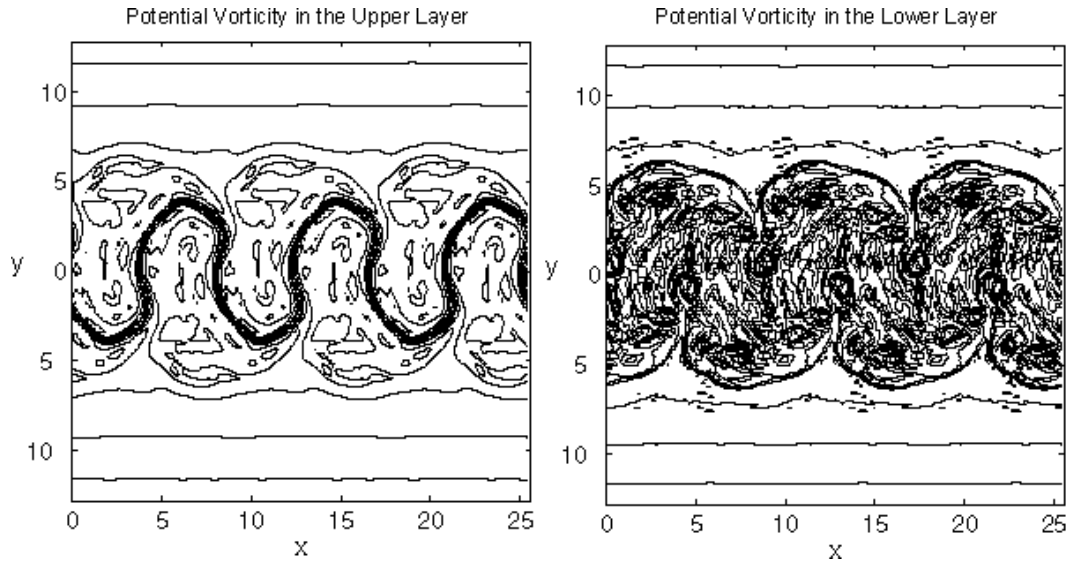
***Figure 1. Frames (a) and (b) show single- and double gyres lying along side an intense boundary current. When a time-dependent wobble of the gyre system is introduced, chaotic advection occurs only in the presence of the twin-gyre geometry. The resulting patterns of dye (Frame c) introduced at the outer edge of the boundary layer, show a fantastic series of folds and filaments in the area occupied by Lagrangian chaos. Other areas remain nonchaotic. Successful Lagrangian prediction is straightforward in the nonchaotic islands. In the chaotic regions, successful Lagrangian prediction is possible only over limited times.***

## RESULTS

(1) The models used for evaluation of turbulent stirring vs. stirring controlled by chaotic advection include a laboratory experiment (Deese *et al.*, 2001, Figures 1 and 2) and various numerical models (Miller *et al.*, 2001 and Yuan *et al.*, 2001, Figure 3). Chaotic advection is associated with persistent organized structures, such as the recirculation gyres shown in Figure 1b or the cat's eye structures in the meandering jet of Figure 3 (left frame). Advection by these features in the presence of time dependence produces limited areas of Lagrangian chaos that advect tracers in extremely complicated ways, leading to distributions of the type shown in Figure 1c. Despite this complexity, Lagrangian prediction (such as in search and rescue) can be quite successful in nonchaotic 'islands' as indicated in Figure 1c. Also, successful Lagrangian prediction can be made over limited time in the chaotic areas of the flow, particularly if invariant manifolds and associated turnstile lobes can be identified. Examples of the latter are shown in Figure 2. The alternative to chaotic advection is conventional turbulence, which is pictured in the right frame of Figure 3. The organized cat's eyes of the upper frame have broken down and formed a complicated, jet-like flow in which invariant manifolds are difficult to define. For most practical purposes, the Lagrangian motion is stochastic and Lagrangian prediction should be carried out accordingly.



***Figure 2. The dye lines are the unstable manifolds of a hyperbolic trajectory lying at the base of the straight line drawn in each frame. The straight line itself is an approximation to a stable manifold. The darkened lobe is a turnstile lobe carrying fluid into the left-hand circulation of Figure 1b. This fluid is in a state of Lagrangian chaos.***



**Figure 3.** *The left frame shows upper-layer potential vorticity profiles from a 2-½-layer model of a meandering jet. The meanders are well organized and Lagrangian motion controlled by chaotic advection. The right frame shows the middle layer potential vorticity for the same run. The flow is disorganized and contains many of the elements of conventional turbulence. Deterministic Lagrangian prediction is possible only for the upper level flow and, even then, is limited in time.*

Using these types of simulations along with results from previous work by other authors, we have identified four measures that distinguish the flows of the type shown in Figure 4 from those of Figures 1–3. The wave-number kinetic energy spectra of the former tends to decay like  $k^{-3}$ , whereas that for the former has a steeper decay. The passive tracer spectra decays like  $1/k$  for conventional turbulence, whereas the spectrum for chaotic advection tends to decay more rapidly. The ratio of Lagrangian to Eulerian time scales for chaotic advection is small; for turbulence it is  $O(1)$ . Finally, Lagrangian dispersion statistics tend to be skewed for chaotic advection and nonskewed for turbulence.

Application of these measures is possible only for very limited data sets. For example, measurements of kinetic wave number spectra in the ocean are limited to altimeter data. The corresponding spectra appear too shallow to support chaotic advection on horizontal scales above 300 km, suggesting that this process is not relevant for basin scale gyres (as has been proposed in a number of recent numerical investigations). From 300 to 50 km, the spectrum lies right at  $k^{-3}$ , indicating a borderline state between chaotic advection and turbulence. Scales smaller than 50 km cannot be resolved.

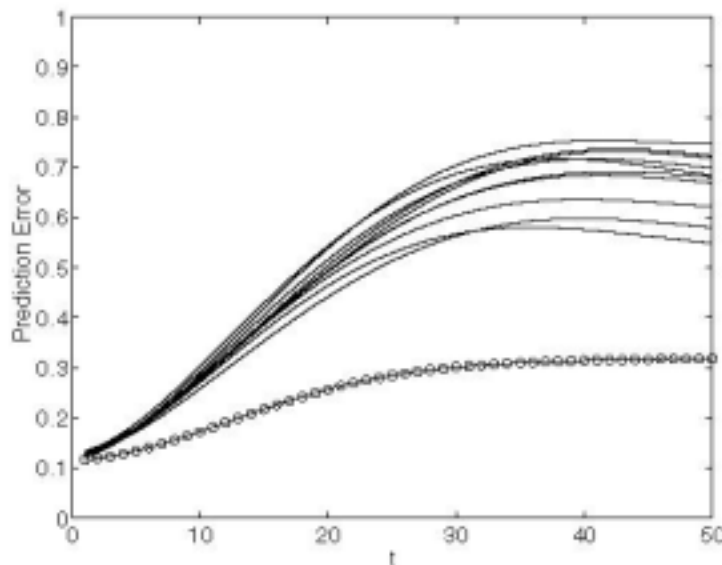
Passive tracer spectra in wave number space are unknown in the published literature, however unpublished work by Rudnick and Ferrari have suggested a  $k^{-2}$  spectrum below the mixed layer in the upper Central North Pacific, a value favorable for chaotic advection.

Field programs involving both float and moored instrumentation have produced Lagrangian to Eulerian time scale ratios. These results indicate conditions favorable for chaotic advection within the intense western boundary currents. No measurements of skewness of float statistics have yet been made but we are encouraging investigators with the means to do so.

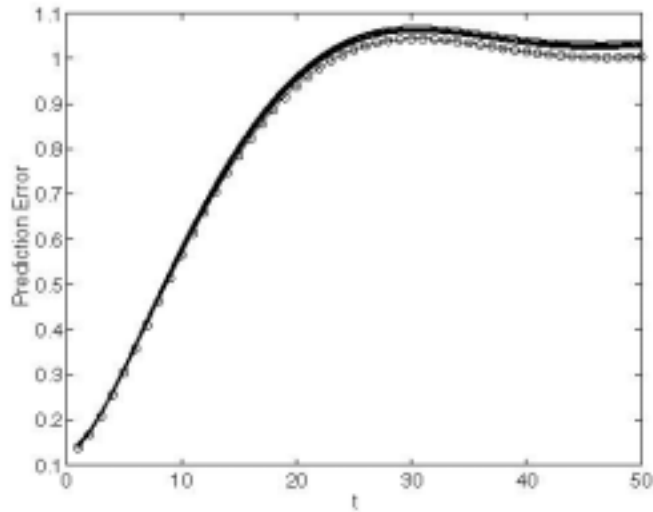
(2) Phase space reconstruction based on aperiodic output from the overflow model has shown that the solutions live on an attractor of dimension 4 or 5. This result is quite encouraging, as the dimension of the model is on the order 10,000. Further information regarding dynamics and predictability follows from a surrogate time series analysis, the results of which are shown in Figure 4. The prediction error based on the surrogate time series (solid curves) are much larger than the prediction errors based on the original (curve with circles), indicating a strong degree of deterministic, nonlinear dynamics in the underlying model. The success of the application to the model has emboldened us to analyze current meter data from the North Carolina shelf and the Middle Atlantic Bight. The shelf data, which is dominated by tides, appears to indicate weakly nonlinear and deterministic origins. Analysis of current meter records from the shelf break region (Figure 5) show very poor predictive capability, the worst cases coming from the moorings closest to the shelf break front. The dynamics may be stochastic or may be deterministic, but with so many degrees of freedom as to render them indistinguishable from stochastic. We are currently analyzing other time series to gain a firmer appreciation for the usefulness of the methods.

## TRANSITIONS

Results on the relevance of chaotic advection in the ocean may influence strategies for search and rescue. Deese's use of dye and particle tracking technology is the first attempt to reconstruct stable and unstable manifolds in a laboratory experiment. These procedures may be of interest to a wide variety of laboratory investigators. The surrogate time series analysis is relatively new to physical oceanography and may be useful to people who analyze time series.



**Figure 4.** *The curve with circles shows the prediction error as a function of time based on a time series of transport generated by a numerical model of a deep overflow. The solid curves give the prediction errors based on surrogate time series formed from the original by randomly varying the phase information but leaving the power spectrum the same. In linear deterministic models (or linear stochastic models) the prediction error for the original should be roughly the same as for the surrogates.*



**Figure 5.** Same as for Figure 4, except that the underlying time series is taken from current meters moored near the shelf break front in the Middle Atlantic Bight.

## RELATED PROJECTS

None.

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